D1-82-0438

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## Stability Diagrams for Magnetogasdynamic Channel Flow

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May 1965

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FLIGHT SCIENCES LABORATORY REPORT NC. 97

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## ABSTRACT

A study is made of the influence of a coplanar mag-
netic field on the stability of a conducting fluid flowing
between parallel planes. After derivation of the general
stability equations for small magnetic Reynolds number,
numerical results are obtained for the case where initial per-
turbations of the magnetic field vanish. This must occur if the
channel walls have zero or infinite conductivity. Four sets of
stability diagrams are presented so that each stability curve
will represent the effect of a given applied magnetic field
as only one of the four quantities in the Reynolds number is
changed. The flow is always stable for initial disturbances of
the field produced by passage of a pulsating current through walls
of finite conductivity.

## NOMENCLATURE

$\vec{B}$      =   magnetic field

$c$      =   wave propagation velocity

$\vec{E}$      =   electric field

$\vec{J}$      =   current

$K$      $= \tilde{\sigma}_w \, \tilde{u}_o \, \tilde{L}$

$N$      =   magnetic interaction parameter $\dfrac{\tilde{\sigma} \, \tilde{B}_o^2 \, \tilde{L}}{\tilde{\rho} \, \tilde{u}_o}$

$R_m$      =   magnetic Reynolds number $\tilde{\sigma} \, \tilde{u}_o \, \tilde{L}$

$R$      -   Reynolds number $\dfrac{\tilde{\rho} \, \tilde{u}_o \, \tilde{L}}{\tilde{\mu}}$

$\alpha$      =   wave number

## Superscripts

$\sim$      =   dimensional quantity

## I. INTRODUCTION

The effect of a uniform coplanar magnetic field on the stability of parabolic flow of a conducting fluid between parallel walls has been the subject of several investigations. The stability equations were first given by Michael[1] who showed Squire's Theorem is also applicable to two-dimensional magnetohydrodynamic flows. In addition to the Reynolds number $R$, two other nondimensional parameters appear in the equations: the magnetic Reynolds number $R_m$, and the interaction parameter $N$.

Because of the complicated form of the stability equations, Stuart[2] simplified the equations by assuming $R_m$ is small. The fluid dynamic and electrodynamic equations are uncoupled because the induced magnetic field is of second order. Stability curves were obtained for constant values of the parameter $q = N\alpha$, where $\alpha$ is the wave number. Unfortunately, the drawbacks of this form as compared to the use of $N$ alone were not recognized by Stuart until the calculations were well under way. The curve $q = .08$ formed a closed loop indicating complete stability above a certain Reynolds number. Stuart claimed the closure of the curve could not be verified because the assumption of small $R_m$ was violated at the higher values of $R$. In a discussion of Stuart's paper, Cowling[3] said the existence of a region of stability for large values of $R$ "hardly seems reasonable".

Using the stability equation for small $R_m$ given by Stuart, Rossow[4] solved the problem again and obtained stability curves for constant values of the parameter $N$. The curves shift to higher $R$ with increased $N$, but do not form closed loops. This suggested the closure of Stuart's curves was due to the use of the parameter $q$ instead of $N$. This

was shown to be the case by Hains[5], who obtained qualitative agreement between the curves of Rossow and Stuart by crossplotting. The agreement is only qualitative because Rossow used the wrong boundary condition along the axis of the channel. The boundary condition applicable to the outer edge of a flat plate boundary layer was used instead of the conditions for an antisymmetric disturbance in a channel. For this reason, Rossow's results are not considered in the remainder of this paper.

Because the existing stability diagrams are of limited value, the stability equation for small $R_m$ has been resolved by an exact numerical method. A useful set of stability diagrams plotted for several non-dimensional parameters is presented in this paper. In addition, this paper attempts to clarify several points which have led to some confusion and misunderstanding about the effects of a magnetic field on stability.

The first point concerns the derivation of the stability equations for small $R_m$. In taking the limit $R_m \to 0$, Stuart neglected a term of the order of the terms retained, and arrived at an equation which is specialized for the case where initial or externally produced perturbations of the magnetic field vanish. This condition is always satisfied by channel walls that are perfect conductors or insulators. Because $R_m$ is small, the field influences the flow to first order, but the flow influences the field only to second order so that first order perturbations of the field can only be generated by passage of an oscillatory current through channel walls of finite conductivity. In an attempt to be more rigorous, Tatsumi[6] introduced a new variable but obtained Stuart's specialized equation. In effect, he also neglected first order perturbations of the magnetic field. In this paper, the general stability equation for small $R_m$ is presented, and solutions are given for channel walls with zero, infinite and finite conductivity.

The second point concerns the choice of parameter used to describe the stability curves, and the correct interpretation of the curves for the particular parameter chosen. In stability problems where the Reynolds number is not the only nondimensional parameter, Hains[7] has shown the change in R should be interpreted as a variation of only those quantities not common to both R and the other nondimensional parameters. For the parameter q chosen by Stuart, this would mean variation of the coefficient of viscosity $\hat{\mu}'$ to change R. Since $R_m$ is independent of $\tilde{\mu}$, the assumption of small $R_m$ is not violated by variation of R in this manner, and the closure of Stuart's curves is therefore correct if $R = R(\tilde{\mu})$. If, as Stuart assumed, $R = R(\tilde{u}_o)$, the closure of the stability curves cannot be decided with the present theory because the assumption of small $R_m$ is violated for large values of R. Since variation of $\tilde{u}_o$ to change R would also change q, some other quantity in q must vary along each of Stuart's stability curves in order to keep q fixed. Clearly, some new parameter independent of $\tilde{u}_o$ must be chosen to describe the stability curves if $R = R(\tilde{u}_o)$. In this paper, parameters are introduced which permit variation of each quantity in R individually.

## II. STABILITY EQUATIONS

The magnetohydrodynamic equations which govern the flow of an incompressible, viscous fluid are (in dimensional form):

flow continuity: $\nabla \cdot \vec{V} = 0$ (1)

Navier-Stokes: $\tilde{\rho} \dfrac{D\vec{V}}{D\tilde{t}} + \nabla \tilde{p} = \vec{J} \times \vec{B} + \tilde{\mu}\nabla^2\vec{V}$ (2)

Ohm's Law: $\vec{J} = \tilde{\sigma}[\vec{E} + \vec{V} \times \vec{B}]$ (3)

Faraday's Law: $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial \tilde{t}}$ (4)

magnetic continuity: $\nabla \cdot \vec{B} = 0$ (5)

Ampere's Law: $\vec{J} = \nabla \times \vec{B}$ (6)

where $\tilde{\mu}$ and $\tilde{\sigma}$ are the viscosity coefficient and electrical conductivity, respectively. $\tilde{p}$ is the fluid pressure, $\tilde{\rho}$ is the density, $\tilde{t}$ is the time and $\vec{E}$ is the electric field. The components of the magnetic field and velocity are, respectively:

$$\vec{B} = \hat{i}\tilde{B}_1 + \hat{j}\tilde{B}_2$$ (7)

$$\vec{V} = \hat{i}\tilde{u} + \hat{j}\tilde{v}$$ (8)

Elimination of $\vec{E}$ from Eqs. (3) and (4) with the aid of (1), (5) and (6) yields

$$\frac{\partial \vec{B}}{\partial \tilde{t}} = \frac{1}{\tilde{\sigma}} \nabla^2\vec{B} + \vec{B}\cdot\nabla\vec{V} - \vec{V}\cdot\nabla\vec{B}$$ (9)

A second equation is obtained by elimination of $\vec{J}$ from Eqs. (2) and (6)

$$\widetilde{\rho} \frac{D\vec{V}}{D\widetilde{t}} + \nabla \widetilde{p} = (\vec{B} \cdot \nabla)\vec{B} - \nabla \widetilde{B}^2 + \widetilde{\mu} \nabla^2 \vec{V} \qquad (10)$$

It is convenient to nondimensionalize by defining the new variables

$$(x,y) = (\widetilde{x},\widetilde{y})/\widetilde{L} \qquad\qquad p = \frac{\widetilde{p}}{\widetilde{\rho}\,\widetilde{u}_0^2}$$

$$(u,v) = (\widetilde{u},\widetilde{v})/\widetilde{u}_0 \qquad\qquad t = \frac{\widetilde{t}\,\widetilde{u}_0}{\widetilde{L}}$$

$$(B_1,B_2) = (\widetilde{B}_1,\widetilde{B}_2)/\widetilde{B}_0 \qquad\qquad N = \frac{\widetilde{\sigma}\,\widetilde{B}_0^2\,\widetilde{L}}{\widetilde{\rho}\,\widetilde{u}_0} \qquad\qquad \Big\} \qquad (11)$$

$$R_m = \widetilde{\sigma}\,\widetilde{u}_0\,\widetilde{L} \qquad\qquad R = \frac{\widetilde{\rho}\,\widetilde{u}_0\,\widetilde{L}}{\widetilde{\mu}}$$

where $\widetilde{L}$ is the channel half-width, and $u_0$ is the velocity at the axis of the channel. The tilda (~) indicates a dimensional quantity.

Eqs. (9) and (10) are linearized by assuming small perturbations in the dependent variable of the form

$$u = U(Y) + \epsilon \frac{\partial\Phi(x,y,t)}{\partial y}$$

$$v = \quad - \epsilon \frac{\partial\Phi(x,y,t)}{\partial x}$$

$$p = P(x,y) + \epsilon \wp'(x,y,t) \qquad\qquad \Big\} \qquad (12)$$

$$B_1 = 1 + \epsilon \frac{\partial\Psi(x,y,t)}{\partial y}$$

$$B_2 = \quad - \epsilon \frac{\partial\Psi(x,y,t)}{\partial x}$$

where $\epsilon$ is a small quantity. These relations automatically satisfy the continuity Eqs. (1) and (5). Assuming solutions of the separable form

$$\phi = \vartheta(y)\exp i\alpha(x - ct) \tag{13}$$

$$\Psi = \psi(y)\exp i\alpha(x - ct) \tag{14}$$

The linearized form of Eqs. (9) and (10) reduce to

$$\frac{i}{\alpha R_m}\left[\psi_{yy} - \alpha^2\psi\right] = \phi - (U - c)\psi \tag{15}$$

$$D\phi = i\alpha N\left[(U - c)\psi - \phi\right] \tag{16}$$

where D is the operator defined by

$$D\phi = (U - c)(\phi_{yy} - \alpha^2\phi) - U_{yy}\phi + \frac{i}{\alpha R}\left[\phi_{yyyy} - 2\alpha^2\phi_{yy} + \alpha^4\phi\right] \tag{17}$$

The steady-state velocity profile is $U = 1 - y^2$. These two coupled equations are the stability equations for plane MHD flow with a uniform coplanar magnetic field. In the remainder of this paper the operator D is used so that the left hand side of Eq. (16) is expressed simply as $D\phi$.

The boundary conditions along the channel walls require the velocity components to vanish

$$\phi(\pm 1) = \phi_y(\pm 1) = 0 \tag{18}$$

If the walls are perfect conductors,

$$\psi(\pm 1) = 0 \tag{19}$$

and if the walls are perfect insulators,

$$\psi(\pm \infty) = 0 \tag{20}$$

For walls of finite conductivity, the value of $\psi(\pm 1)$ takes on some unknown value found by matching the solution in the channel to the solution in the wall.

Because of the complicated form of Eqs. (15) and (16), we restrict our attention to the special case of small $R_m$. Assuming

$$\psi = \psi_o + R_m \psi_1 + \cdots$$

$$\phi = \phi_o + R_m \phi_1 + \cdots \quad\quad\quad (21)$$

Eqs. (15) and (16) become

$$\psi_{oyy} - \alpha^2 \psi_o = 0 \quad\quad\quad (22)$$

$$D\phi_o = i\alpha N \Big[ (U-c)\psi_o - \phi_o \Big] \qu\quad\quad (23)$$

This is the general form of the stability equations for small $R_m$.

Stuart[2] obtained a single equation which corresponds to Eq. (23) without the $\psi_o$ term. It is therefore specialized for the case where the perturbations in the magnetic field of the order of $\psi_o$ vanish. Instead of using Eq. (15), Stuart performed an order of magnitude analysis on the equation

$$\frac{1}{\alpha R_m} (\tilde{\psi}_{yy} - \alpha^2 \tilde{\psi}) = \frac{\tilde{B}_o}{\tilde{U}_o} \tilde{\phi} - (U-c)\tilde{\psi} \qu\quad\quad (24)$$

where $\tilde{\psi} = B_o \psi$ and $\tilde{\phi} = \tilde{U}_o \phi$ are __dimensional__ quantities. By assuming the terms $\tilde{\psi}$ and $\tilde{\phi}$ instead of $\psi$ and $\phi$ were of the same order he neglected the $\tilde{\psi}$ term on the right hand side of Eq. (23) in comparison with the other terms. Eqs. (22) and (23), obtained by assuming $\psi_o$ and $\phi_o$ are of the same order, reduce to Stuart's stability equation when $\psi_o = 0$.

In an attempt to be more rigorous in the derivation of the stability equations, Tatsumi[6] introduced a new variable

$$\eta = \psi_{yy} - \alpha^2 \psi \qquad (25)$$

and used Eq. (15) in the form

$$\phi = \frac{i}{\alpha R_m} \eta + (U - c)\psi \qquad (26)$$

to eliminate $\phi$ from Eq. (16). The resultant equation is

$$D\eta + i\alpha N\eta = i\alpha(U - c)R_m D\psi \qquad (27)$$

For small $R_m$, Tatsumi neglected the right hand side of Eq. (27) and obtained an equation of the form of Eq. (29) below. He proceeded to outline the solution of the differential equation for the boundary conditions $\eta = \eta' = 0$ at both boundaries. It is easy to show, in the limit as $R_m$ that Tatsumi's equation is satisfied by $\eta = 0$. By using Eq. (21) and a similar power series expansion for $\eta$, Eqs. (26) and (27) are, to first order,

$$\eta_0 = 0 \qquad (28)$$

$$D\eta_0 + i\alpha N\eta_0 = 0 \qquad (29)$$

The corresponding second order equations are

$$\phi_0 = \frac{i\eta_0}{\alpha} + (U - c)\psi_0 \qquad (30)$$

$$D\eta_1 + i\alpha N\eta_1 = i\alpha(U - c)D\psi_0 \qquad (31)$$

It is clear that Tatsumi's equation is not equivalent to Stuart's stabil

equation because the variable $\eta$ vanishes for zero $R_m$. Much of the confusion results because Eq. (29) and Eq. (31) with $\psi_0 = 0$ have the same form as Eq. (23) with $\psi_0 = 0$. These equations are by no means equivalent. Tatsumi's equation is meaningful only for small values of $R_m$ greater than zero, while Stuart's equation is valid for $R_m = 0$ as well. Both stability equations assume $\psi_0 = 0$, which is a special case.

### III.  CHANNEL WITH WALLS OF FINITE CONDUCTIVITY

If the walls have finite conductivity $\tilde{\sigma}_w$, initial perturbations of the magnetic field can be induced by passage of a pulsating current $\tilde{J}_z$ through the walls.  In the walls, which extend to $y = \pm \infty$, the magnetic field must satisfy Eq. (9) with $\vec{V} = 0$.  Using Eq. (12) for the field components $B_1$ and $B_2$, and Eq. (14) for the form of the disturbance, Eq. (9) reduces to the form

$$\psi_{oyy} + \beta^2 \psi_o = 0 \tag{32}$$

where

$$\beta^2 = -\alpha^2 + i\alpha Kc \tag{33}$$

This equation applies only when $R_m$ is small.  In addition, the nondimension parameter $K = \tilde{\sigma}_w \, \tilde{u}_o \, \tilde{L}$ is assumed to be of order one.

If the disturbances are symmetric with respect to the channel axis, the solutions of Eqs. (22) and (32) are, respectively,

$$0 \leq y \leq 1 \qquad \psi_o = A \cosh \alpha y \tag{34}$$

$$y \geq 1 \qquad \psi_o = B \cos \beta y \tag{35}$$

The boundary condition requiring the matching of $\psi_o$ and $\psi_{oy}$ at $y = \pm 1$ leads to the relation

$$\alpha \tanh \alpha = -\beta \tan \beta \tag{36}$$

Since $\alpha$ is real and positive, $\beta$ must also be real.  Eq. (33) shows $c$ must be a negative imaginary number since $K$ is positive.  The disturbances are always stable.

## IV. CHANNEL WITH WALLS OF ZERO OR INFINITE CONDUCTIVITY

If the channel walls are perfect conductors ($\widetilde{\sigma}_w = \infty$) or perfect insulators ($\widetilde{\sigma}_w = 0$), initial perturbations in the magnetic field must vanish. $\psi_0$ can also vanish for walls of finite conductivity if the current $\widetilde{J}_z$ is zero. The eigenvalue problem reduces to the solution of Eq. (23) with $\psi_0 = 0$. The boundary conditions are given by Eq. (18). As is customary in the nonmagnetic case, we restrict our analysis to antisymmetrical disturbances so that $\phi$ is an even function of y. Integration is only necessary over half the channel if the boundary condition

$$\phi_y = \phi_{yyy} = 0 \tag{37}$$

is satisfied at the channel axis.

The problem, as it is presently posed, was first solved by Stuart[2] using the Tollmien-Schlichting theory. In this paper, solutions are obtained by an exact numerical method developed by Hains and Price[8] for Poiseuille flow between flexible walls. Details of the numerical method can be found in Reference 8, but a brief outline will be given here.

The numerical solution is begun by dividing the channel half-width into n parts of equal length. After introduction of a new dependent variable developed to reduce truncation errors, the differential equation and boundary conditions are written in finite difference form at each of the divisions. The resultant system of linear algebraic equations possesses a nontrivial solution if, and only if, the determinant of the coefficients vanishes. This condition determines the eigenvalue c. For each combination of $\alpha$, R, N specified, three initial guesses for c are made, and the value of the determinant for each guess is calculated. A complex quadratic is passed through these three values, and the root of the quadratic nearest

the last guess is taken as the new approximation to the eigenvalue. For n = 50, iteration was continued until the value of the determinant had been reduced to some preselected value close to zero. The final value of c is the eigenvalue. This numerical method was programmed for an IBM 7090 computer using single precision with 8 digits. The values of c given in this paper are accurate to at least four decimal places. This accuracy was checked by repeating the calculation of some points with a double precision program carrying 16 digits.

Neutral stability curves for the wave number $\alpha$ and wave propagation velocity $c_r$ are shown in Figs. 1-6. Three sets of curves are presented for the three nondimensional parameters N, N′ and N*, where

$$
\begin{aligned}
N' &\equiv NR &= \tilde{\sigma}\, \tilde{B}_o^2\, \tilde{L}/\tilde{\mu} \\
N* &\equiv N/R &= \tilde{\sigma}\, \tilde{\mu}\, \tilde{B}_o^2/(\tilde{\rho}\, \tilde{u})^2
\end{aligned}
\right\} \quad (38)
$$

Since N is independent of $\tilde{\mu}$, these parameters have been selected so that N′ is independent of $\tilde{\rho}$ and $\tilde{u}_o$, and N* is independent of $\tilde{L}$. Use of all three parameters to describe the stability curves permits the variation of R to be interpreted as a change in one of the quantities $\tilde{\rho}$, $\tilde{u}_o$, $\tilde{L}$ or $\tilde{\mu}$.

In Figures 1 and 2 where N has been chosen as the interaction parameter, a given value of the applied magnetic field will lead to a fixed value of N if the viscosity is varied to change R. This is indicated by $R = R(\tilde{\mu})$ along the abscissa of the figure. As the figures clearly show, the critical Reynolds number increases with N and the wave propagation velocity $c_r$ decreases with N.

In a channel flow experiment, it is customary to increase the Reynolds number by increasing the flow velocity $\tilde{u}_o$. Stuart also interpreted
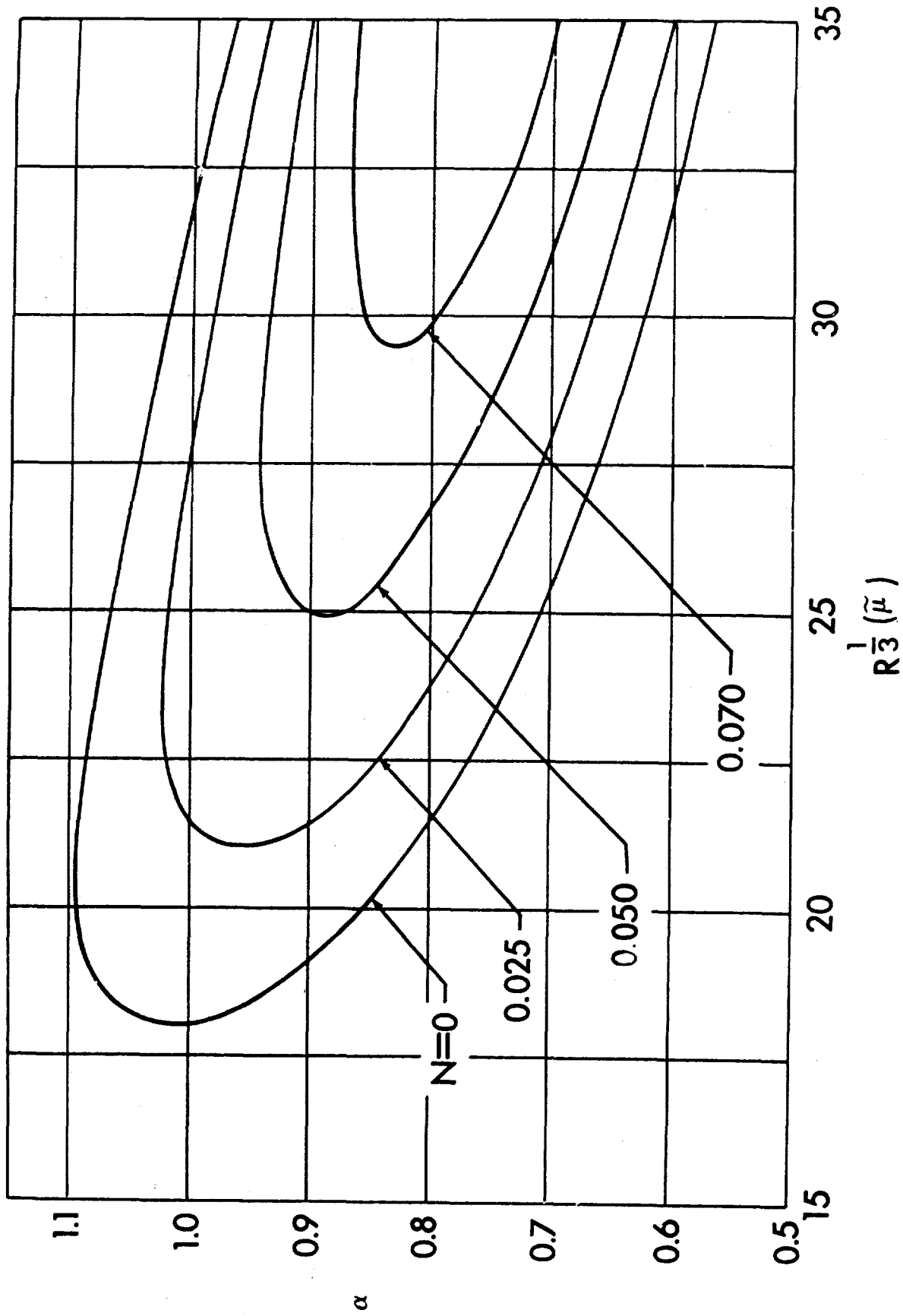
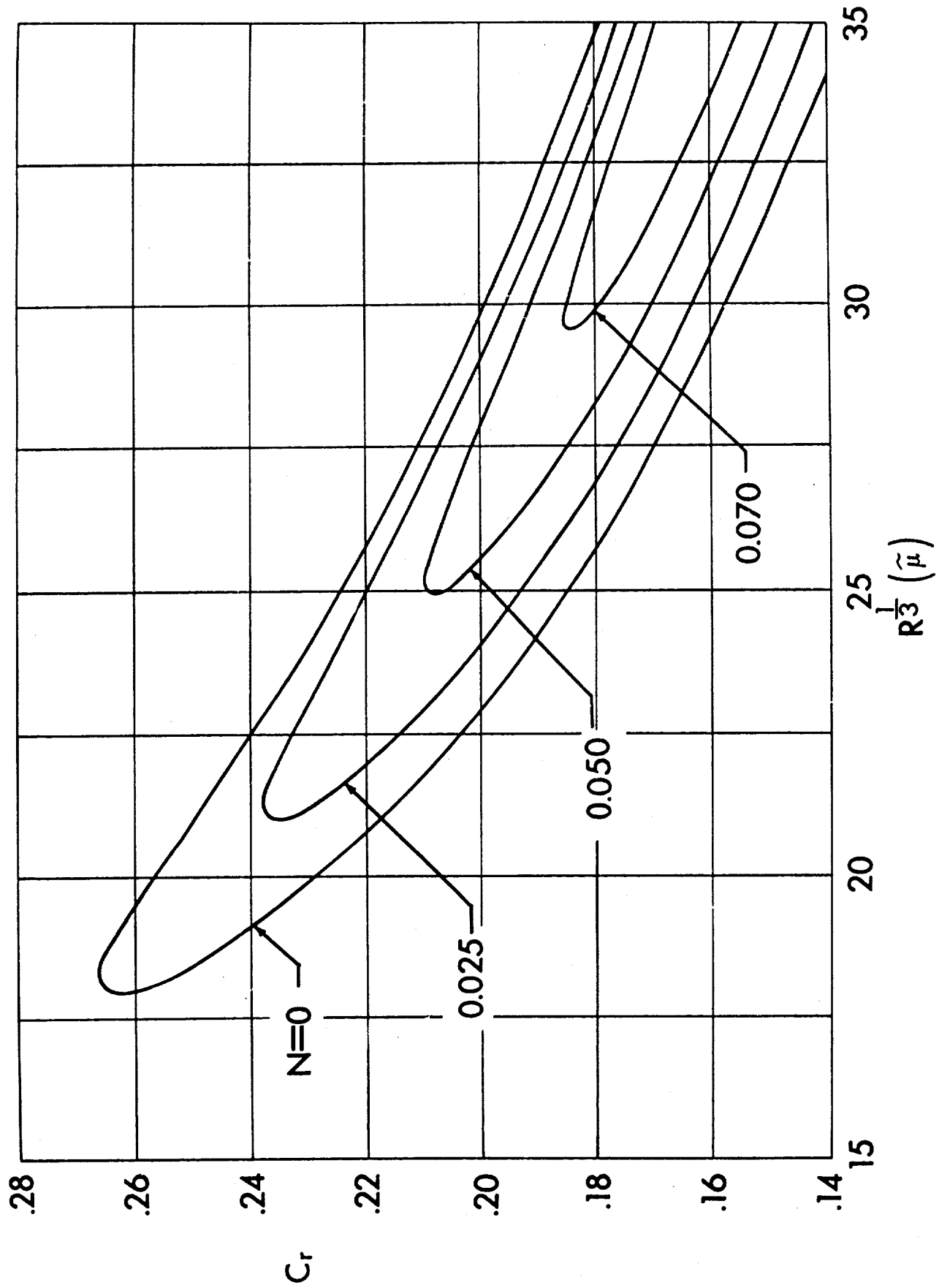Fig. 1    Neutral stability curves for the wave number α for various
values of N.

Fig. 2   Neutral stability curves for the wave propagation speed $c_r$
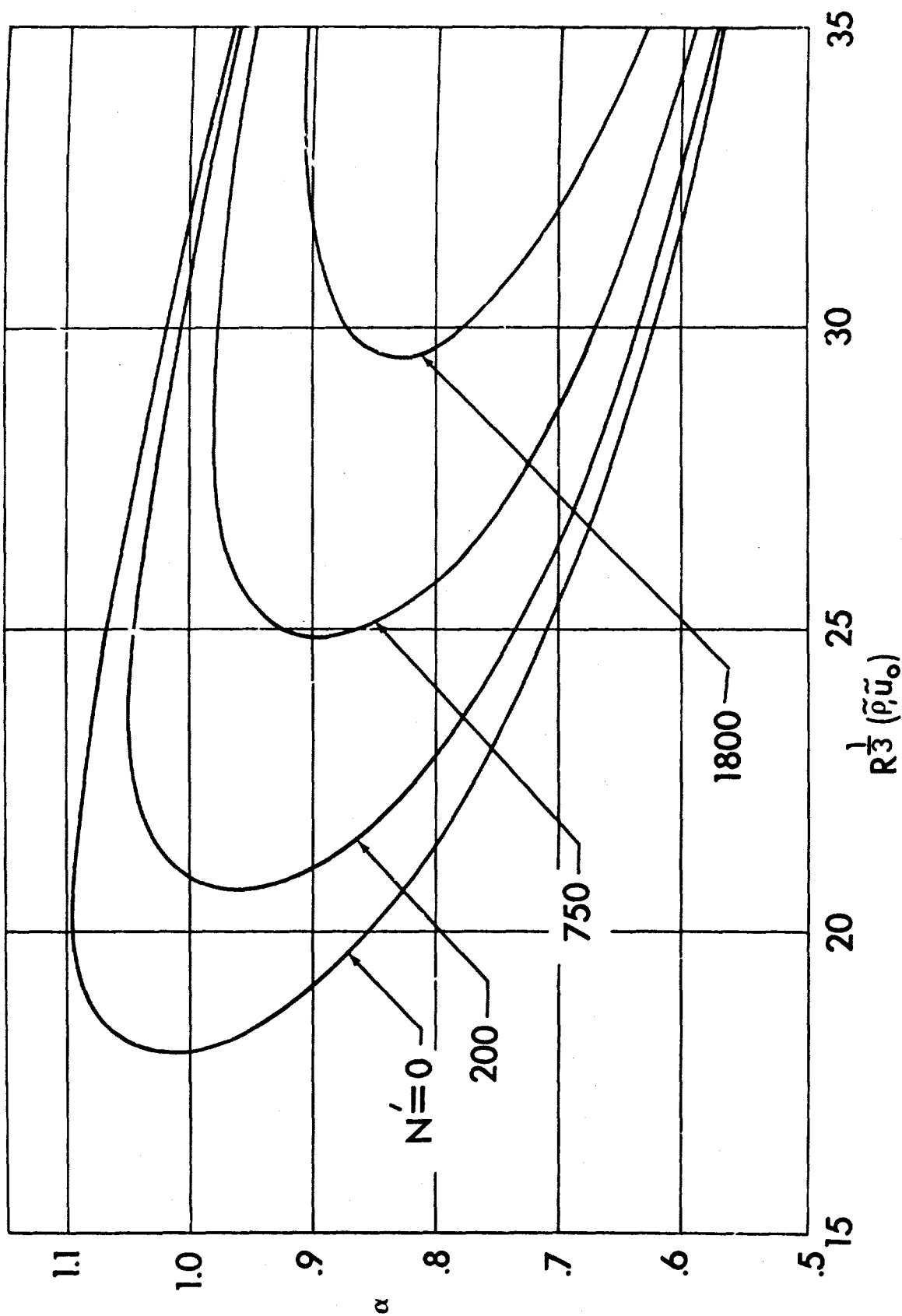
for various values of N.

Fig. 3    Neutral stability curves for the wave number $\alpha$ for various values of $N'$.

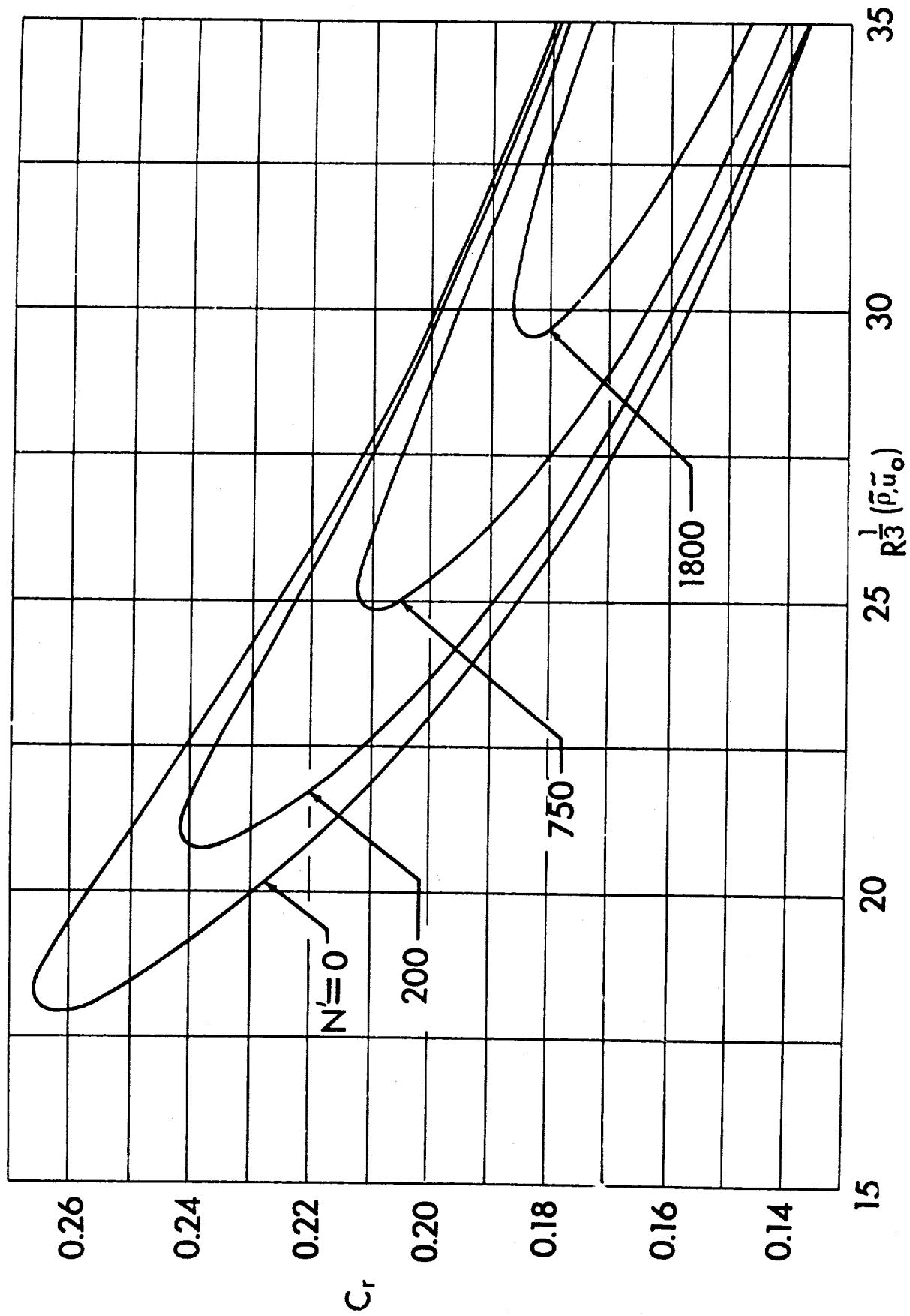Fig. 4   Neutral stability curves for the wave propagation speed $c_r$

for various values of N'.
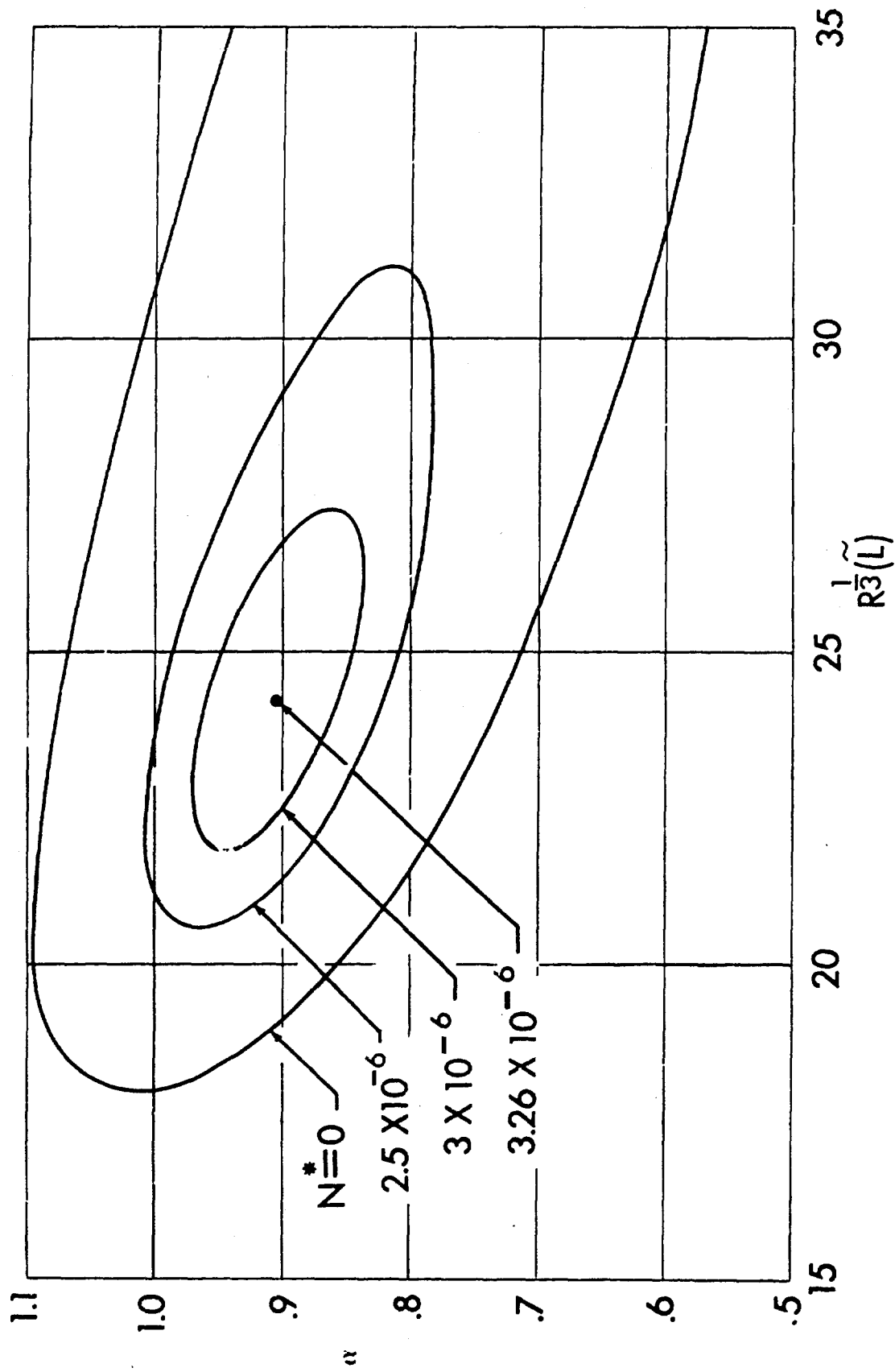
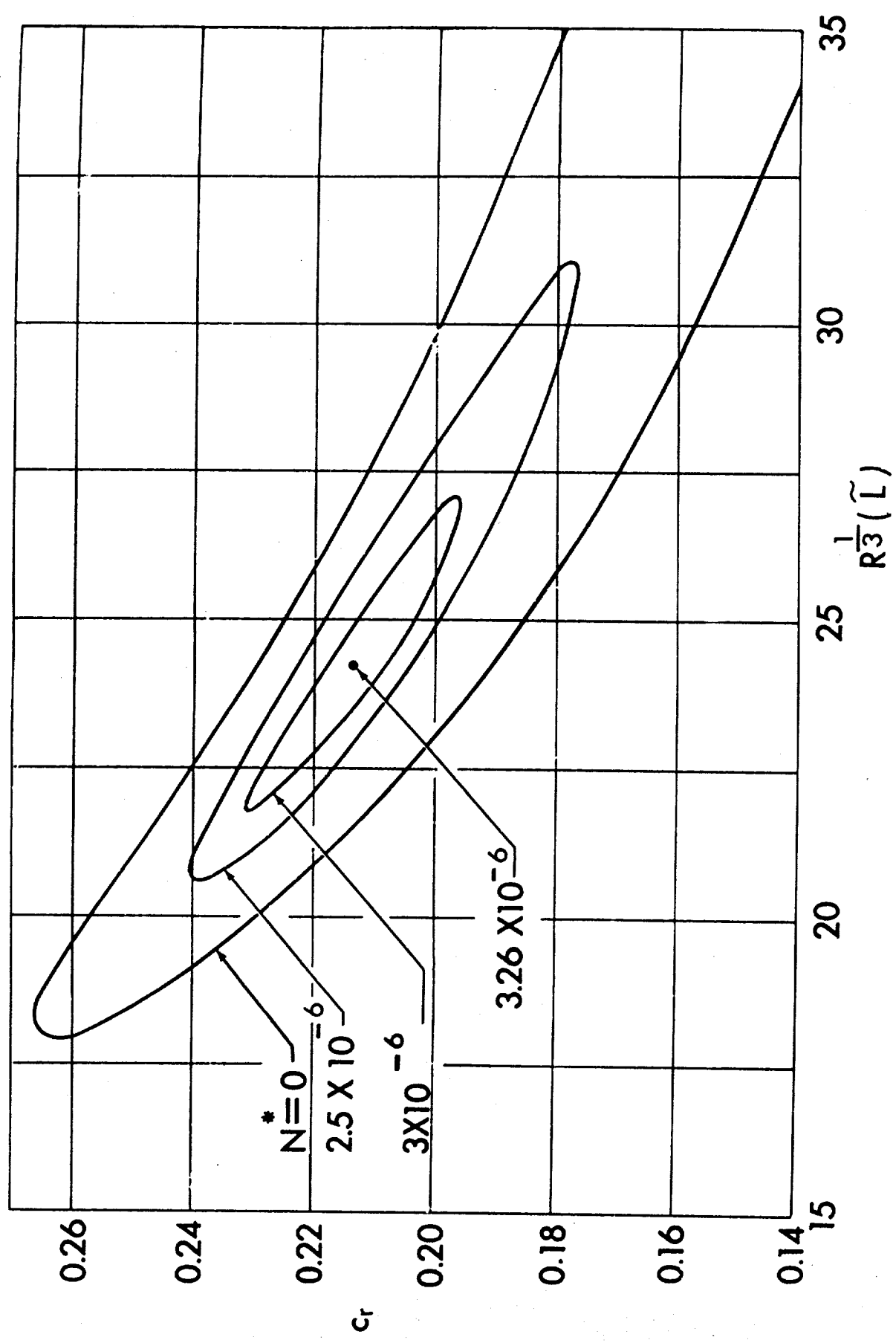Fig. 5    Neutral stability curves for the wave number α for various

values of N*.

Fig. 6    Neutral stability curves for the wave propagation speed $c_r$ for various values of $N^*$.

his stability curves in the same manner. Instead of using N, or N$\alpha$ as Stuart used to describe the curves, the parameter N' was chosen because of its independence of $\tilde{u}_o$. The stability curves are shown in Figs. 3 and 4. The curves are similar to those for N, but approach the nonmagnetic curve N' = 0 more rapidly with increased R($\tilde{\rho}$, $\tilde{u}_o$). Numerical values of c over a range in $\alpha$ and R$^{1/3}$ are given in Table I for some representative values of N'. Positive values of the imaginary part of c indicate an unstable disturbance.

The last set of curves shown in Figs. 5 and 6 show the effect of variation of the channel width $\tilde{L}$ to change the Reynolds number. As N* is increased, the neutral stability curves form closed loops with the region of instability confined to the interior of the loop. This means the flow is stable for small channel widths and for large channel widths. When N* > 3.26 $\times$ 10$^{-6}$, the region of instability disappears and the flow is stable for all channel widths.

The distribution of the eigenfunction is shown in Figure 7. Only the real part of $\phi$ and its first derivative are shown because the imaginary parts are small in comparison. The magnetic field tends to reduce the gradients of the velocity perturbations in the region near the wall. Using the eigenfunction for N = 0, the streamline distribution shown in Figure 8 was calculated. For clarity, the bending of the streamlines was greatly exaggerated by taking $\epsilon$ = 1. The antisymmetric form of the disturbance is clearly evident from this figure.
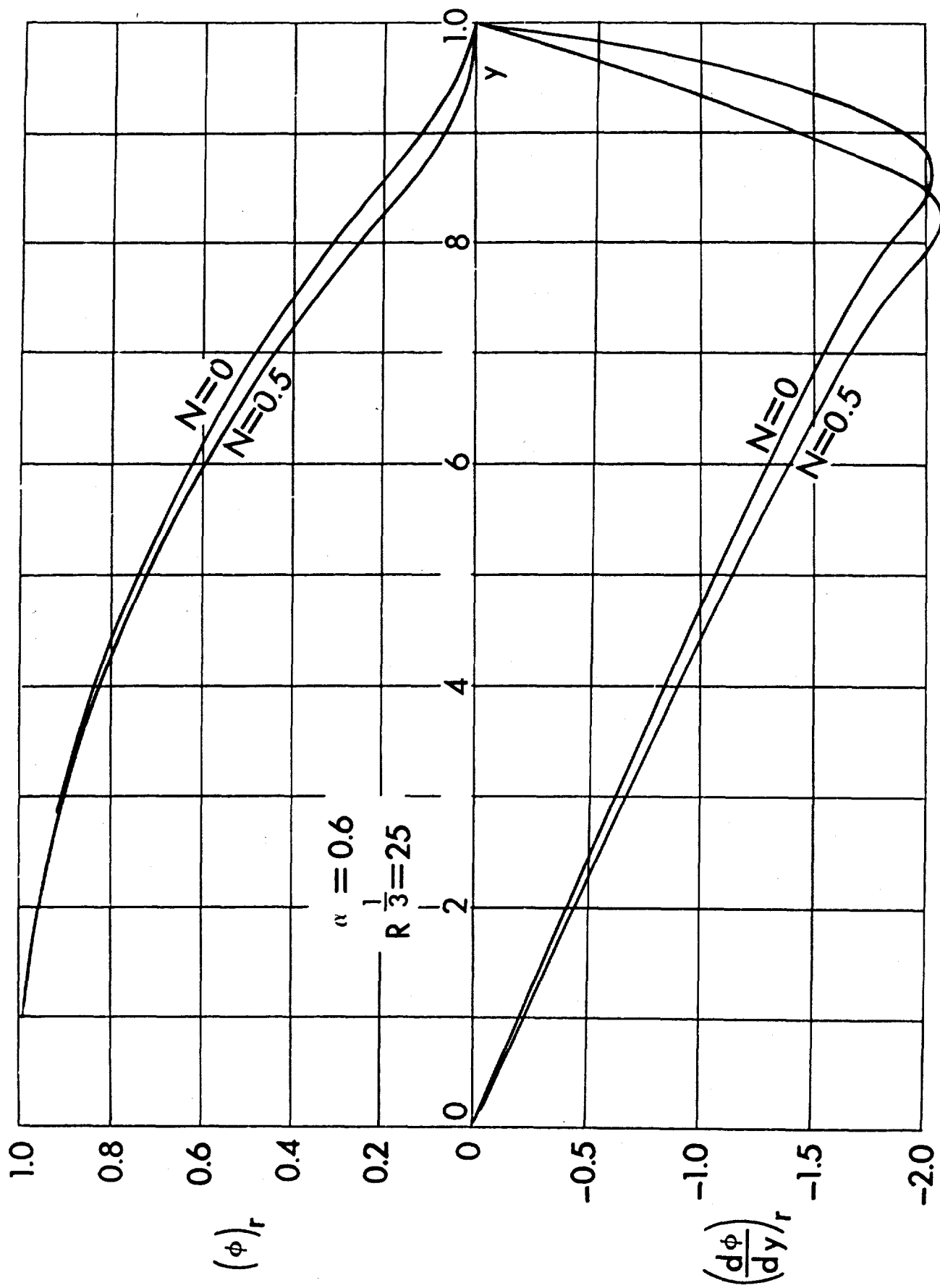
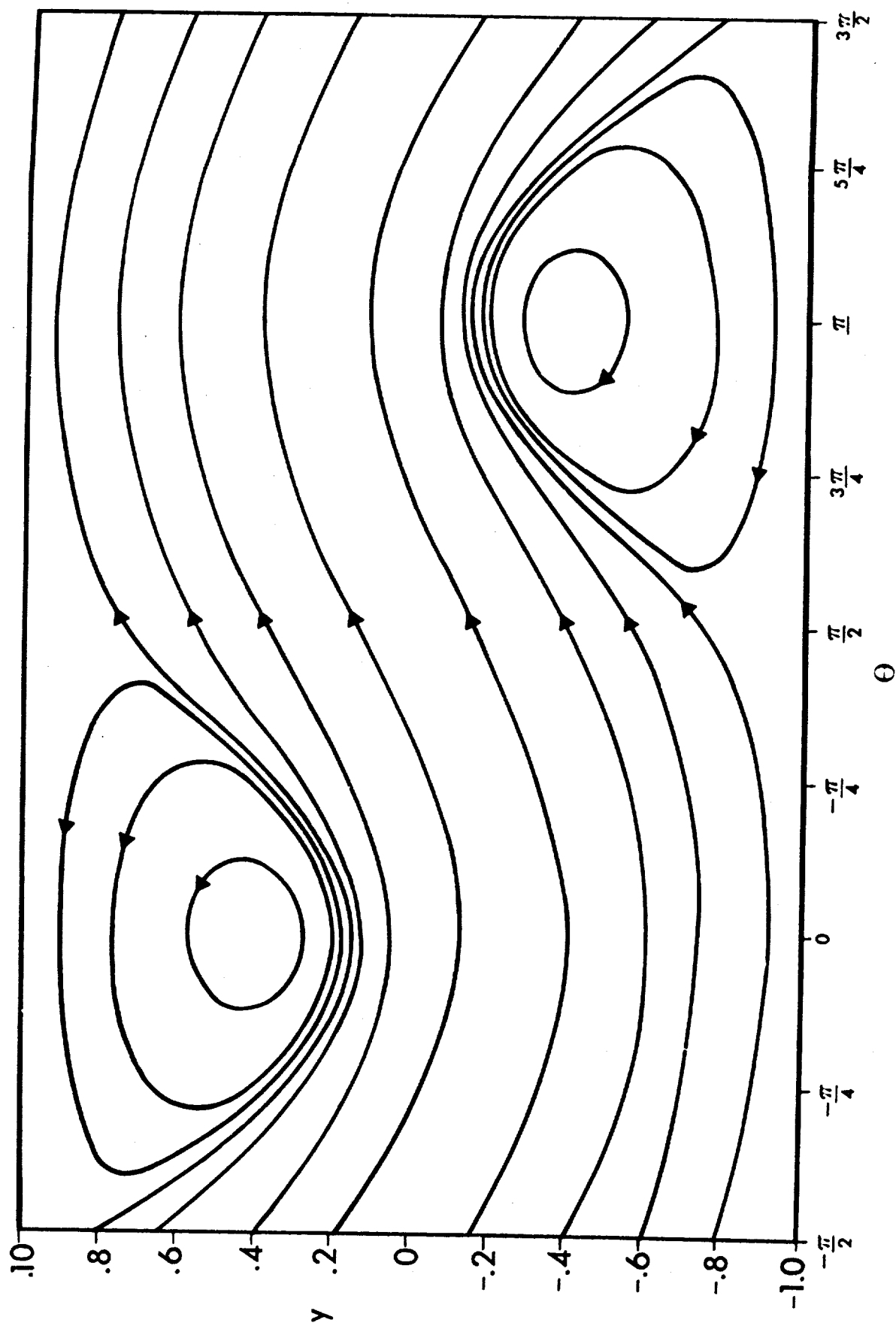Fig. 7   Variation of the real part of the eigenfunction $\phi$ and its derivative $\frac{d\phi}{dy}$ .

Fig. 8    Streamline pattern over a wavelength in $\theta = i\alpha(x-ct)$, $\alpha = 0.5$,

$R = (25)^3$, $\epsilon = 1.0$, $N = 0$.

## V. CONCLUSIONS

A study has been made of the stability of a conducting fluid between two parallel planes with an aligned magnetic field. When the magnetic Reynolds number is small, three different methods of producing a disturbance are possible. The initial disturbance can be in the fluid, in the magnetic field, or in both simultaneously.

When the channel walls have finite conductivity, a disturbance of the magnetic field can be introduced by passage of an oscillatory current through the walls. This in turn causes the fluid to oscillate, but this type of disturbance is always stable.

Neutral stability curves have been presented for initial disturbances produced only in the fluid. The effect of the magnetic field is to increase stability, but the particular shape of the stability curves depends on how the Reynolds number is varied. Four sets of stability curves were presented, each corresponding to a variation of one of the four quantities in the Reynolds number. The stability curves form closed loops when the channel width is varied to change the Reynolds number. The loops disappear and complete stability is obtained when $N^* \geq 3.26 \cdot 10^{-6}$. When other quantities are varied to increase the Reynolds number, the stability curves are shifted to higher Reynolds number as the interaction parameter is increased.

TABLE I

N' = 0

| a/R$^{1/3}$ | 16 | 20 | 22 | 24 | 26 | 28 | 30 |
|---|---|---|---|---|---|---|---|
| .60 | .1955 − .0037i | .1890 − .0259i | .1803 − .0192i | .1725 − .0136i | .1655 − .0091i | .1592 − .0054i | .1534 − .0023i |
| .65 | .2055 − .0266i | .1961 − .0188i | .1888 − .0125i | .1804 − .0075i | .1730 − .0035i | .1662 − .0003i | .1601 + .0023i |
| .70 | .2174 − .0199i | .2065 − .0127i | .1966 − .0069i | .1878 − .0025i | .1800 + .0009i | .1729 + .0036i | .1666 + .0057i |
| .75 | .2291 − .0142i | .2143 − .0075i | .2039 − .0024i | .1947 + .0014i | .1866 + .0042i | .1793 + .0063i | .1728 + .0078i |
| .80 | .2335 − .0092i | .2216 − .0033i | .2108 + .0010i | .2013 + .0042i | .1929 + .0064i | .1855 + .0079i | .1788 + .0088i |
| .85 | .2413 − .0054i | .2285 − .0002i | .2174 + .0033i | .2076 + .0057i | .1990 + .0073i | .1913 + .0082i | .1844 + .0096i |
| .90 | .2483 − .0025i | .2351 + .0017i | .2236 + .0045i | .2136 + .0063i | .2047 + .0072i | .1966 + .0075i | .1897 + .0073i |
| .95 | .2545 − .0005i | .2413 + .0027i | .2295 + .0047i | .2191 + .0058i | .2100 + .0060i | .2018 + .0057i | .1944 + .0049i |
| 1.00 | .2611 + .00005i | .2471 + .0027i | .2349 + .0039i | .2242 + .0042i | .2147 + .0038i | .2051 + .0029i | .1953 + .0015i |
| 1.05 | .2665 − .00012i | .2524 + .0017i | .2398 + .0021i | .2267 + .0017i | .2157 + .0006i | .2097 − .00096i | .2013 − .0029i |
| 1.10 | .2722 − .0011i | .2573 − .0002i | .2441 − .0005i | .2325 − .0016i | .2219 − .0034i | .2121 − .0056i | .2030 − .0080i |
| 1.15 | .2770 − .0029i | .2614 − .0026i | .2477 − .0039i | .2353 − .0057i | .2240 − .0061i | .2133 − .0107i | .2031 − .0134i |

N' = 750

| a/R$^{1/3}$ | 20 | 22 | 24 | 26 | 28 | 30 |
|---|---|---|---|---|---|---|
| .65 | .2144 − .0287i | .2018 − .0182i | .1913 − .0107i | .1823 − .0052i | .1746 − .0011i | .1617 − .0015i |
| .70 | .2211 − .0226i | .2082 − .0133i | .1975 − .0066i | .1864 − .0016i | .1805 + .0016i | .1677 + .0019i |
| .75 | .2271 − .0181i | .2141 − .0096i | .2033 − .0037i | .1941 + .0004i | .1861 + .0033i | .1736 + .0041i |
| .80 | .2328 − .0146i | .2197 − .0071i | .2088 − .0019i | .1995 + .0015i | .1915 + .0037i | .1791 + .0052i |
| .85 | .2380 − .0124i | .2249 − .0056i | .2140 − .0012i | .2046 + .0016i | .1964 + .0032i | .1843 + .0051i |
| .90 | .2429 − .0111i | .2296 − .0051i | .2186 − .0015i | .2092 + .0006i | .2008 + .0016i | .1891 + .0040i |
| .95 | .2474 − .0107i | .2342 − .0056i | .2230 − .0027i | .2133 − .0013i | .2046 − .0010i | .1933 − .0018i |
| 1.00 | | | | | | .1968 − .0014i |

N' = 1800

| $a/R^{1/3}$ | 20 | 22 | 24 | 26 | 28 | 30 | 33 | 35 |
|---|---|---|---|---|---|---|---|---|
| .60 | | | | | | | .1487 - .0053i | .1433 - .0022i |
| .65 | | | | | | | .1544 - .0012i | .1490 - .0014i |
| .70 | .2285 - .0467i | .2105 - .0319i | .1969 - .0210i | .1861 - .0132i | .1771 - .0074i | .1695 - .0030i | .1599 - .0016i | .1544 + .0037i |
| .75 | .2300 - .0401i | .2153 - .0265i | .2019 - .0167i | .1912 - .0096i | .1823 - .0045i | .1747 - .0007i | .1651 + .0031i | .1596 + .0046i |
| .80 | .2354 - .0351i | .2196 - .0224i | .2066 - .0135i | .1951 - .0072i | .1872 - .0027i | .1797 + .0005i | .1701 + .0035i | .1645 + .0047i |
| .85 | .2401 - .0314i | .2235 - .0197i | .2110 - .0116i | .2006 - .0059i | .1916 - .0021i | .1843 + .0021i | .1746 + .0027i | .1690 + .0034i |
| .90 | .2434 - .0288i | .2275 - .0180i | .2151 - .0106i | .2048 - .0057i | .1961 - .0025i | .1885 - .0005i | .1787 + .0009i | .1729 + .0011i |
| .95 | .2465 - .0271i | .2311 - .0171i | .2188 - .0106i | .2065 - .0063i | .1996 - .0038i | .1921 - .0024i | .1820 - .0018i | .1761 - .0022i |

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